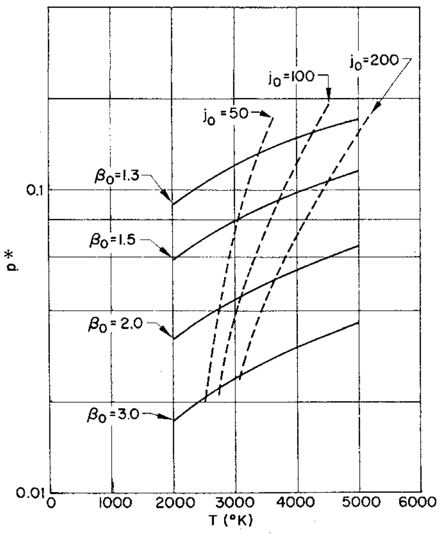

Fig. 9 Optimum pressure ratios for I_{sp} = 2500 sec, helium seeded with 1% cesium, $p_0 = 10^5$ newtons/m², M_0 = 1, and B = 1 weber/m² with lines of constant maximum current density, j_0, in amp/cm².

It corresponds to $\beta_0 \simeq 1.5$, for which A^* (Fig. 4) varies only slightly from unity. If a lower exit pressure is acceptable, however, β_0 can be chosen larger and an expanding channel can be designed which is much shorter. This would require higher currents, but not as high as the constant area case, since it is found that the maximum current required by the constant area case is about three times higher than the optimum.

Conclusions

A numerical solution has been obtained, which, for given inlet and outlet conditions and constant magnetic field determines the minimum length isothermal crossed field accelerator. Although the solution does not exhaust all of the possibilities for isothermal acceleration, it demonstrates that, for the interesting case of constant magnetic field, considerable reductions in length are possible. The results easily can be extended to other field shapes and can be refined to include the variation of electrical conductivity with pressure and current density.

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Nonequilibrium-Dissociated Boundary Layers with a Reacting Inviscid Flow

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This paper examines the problem of nonequilibrium boundary layers when chemical reaction occurs along the inviscid flow streamlines at the outer edge for the case of high Reynolds number. A generalized species conservation equation in similarity coordinates, which explicitly shows the influence of nonequilibrium inviscid flow reaction on the boundary layer, is derived and discussed. Except in the special case of a highly cooled body immersed in a slow, hot, dissociatively relaxing gas stream, it is shown that the inviscid reaction rate can be neglected in analyzing nonequilibrium boundary-layer flows encountered in practice, unless the details of the gas-state profiles near the outer edge of the boundary layer are of interest.

Nomenclature

α = atom mass fraction
$\bar{c}_p$ = frozen specific heat of mixture
η = similarity coordinate, Eq. (4)
f = stream function in similarity coordinates
g = net reaction rate function, Eq. (2)
Γ = characteristic (flow time/reaction time) parameter, Eq. (8)
G,H = nondimensional reaction rate distribution functions, Eqs. (5–7)
h_D = specific dissociation energy of molecules
κ = $x(d\ln\xi/dx)$
k_R' = recombination rate constant
μ = coefficient of viscosity
ω = recombination rate temperature-dependence exponent
p = static pressure
ρ = mixture density
r_B = local body radius
$\mathcal{R}$ = universal gas constant
ξ = similarity coordinate, Eq. (4)
θ = T/T_e

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θ_D = T_D/T_e
T = absolute temperature
T_D = dissociation temperature of molecules
u,v = velocity components parallel and normal to body, respectively
x,y = coordinates parallel and normal to body, respectively
z = α/α_e

Subscripts

e = conditions at edge of boundary layer
eq = equilibrium flow
F = chemically frozen flow
w = wall conditions

Introduction

NONEQUILIBRIUM-DISSOCIATED laminar boundary-layer flows have been studied theoretically for a variety of body shapes ranging from blunt-nosed configurations[1–5] to a simple, flat-plate geometry.[6, 7] In all of these analyses, the inviscid flow at the edge of the boundary layer is taken to be either nonreacting or in a state of complete equilibrium; the possible effects of a simultaneously reacting, nonequilibrium inviscid flow and boundary layer have not been considered. However, this question can arise in connection with either sharp or blunt-nosed aerodynamic bodies in hypersonic free flight at high altitudes where both the viscous and inviscid portions of the flow field can be appreciably out of equilibrium. The question also arises when such bodies are immersed in a reacting gas stream produced by an advanced hypersonic wind tunnel or shock tube. Moreover, the boundary layer on the nozzle or tube wall itself in these devices can develop in the presence of a nonequilibrium-dissociating inviscid flow.

This paper examines the effect of chemical reaction in the inviscid flow on the chemical state within the boundary layer in such problems for the case of high Reynolds number flows where vorticity interactions are negligible. Specifically, it will be the object to determine when and why the inviscid reaction rate may be neglected in analyzing nonequilibrium boundary-layer flows. For this purpose, a generalized species conservation equation in similarity coordinates, which explicitly shows the influence of nonequilibrium inviscid flow reaction, is derived and discussed for the representative case of a binary, dissociating diatomic gas mixture. The relative significance of the inviscid reaction term will then be examined for both low-speed, highly cooled flows and high-speed flows with large viscous dissipation heating.

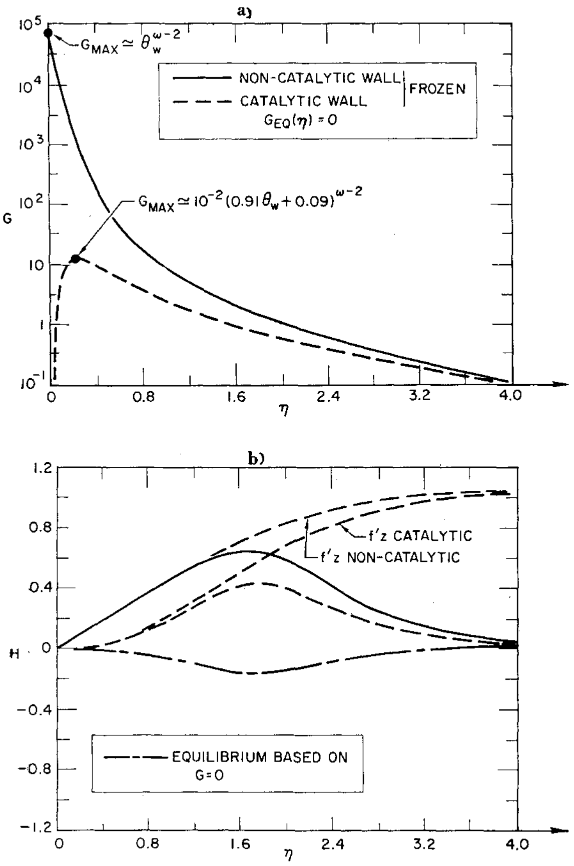

Fig. 1 Reaction rate distribution functions G and H for highly cooled, nondissipative flow; $\theta_w = 0.04$, $U_e^2/2\bar{c}_pT_e = 0$, $\theta_D = 10$, $\alpha_e = 0.50$, $P_R = L_e = 1$.

Species Conservation Equation

Consider laminar boundary-layer flow of a dissociated diatomic gas over a two-dimensional or axially symmetric body at a given temperature T_w whose surface is either completely catalytic or noncatalytic with respect to heterogeneous atom recombination. To simplify the discussion without sacrificing the essential features of interest, the Prandtl and Schmidt numbers are assumed to be unity and the product of density and viscosity coefficient is taken to be constant across the boundary layer. Then the effect of nonequilibrium chemical reaction on the boundary layer is described by the atom mass conservation equation[1]:

$$u\frac{\partial\alpha}{\partial x} + v\frac{\partial\alpha}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial\alpha}{\partial y}\right) - \frac{\dot{w}}{\rho} \tag{1}$$

where

$$\frac{\dot{w}}{\rho} = 4k_R{}'T^{\omega-2}\left(\frac{p_e}{\mathcal{R}}\right)^2 g \equiv 4k_R{}'T^{\omega-2}\left(\frac{p_e}{\mathcal{R}}\right)^2 \times \left[\frac{\alpha^2}{1+\alpha} - \frac{(1-\alpha)\exp[A-(T_D/T)]}{4p_e}\right] \tag{2}$$

is the net recombination-dissociation rate in the gas, $k_R{}'T^\omega$ is the recombination rate "constant," T_D is the characteristic dissociation temperature, A is a constant (≈ 16), and the remaining symbols are defined in the nomenclature. The funtion $g(\alpha, p_e, T)$, defined by the bracketed quantity in Eq. (2), has the important property of vanishing identically at chemical equilibrium. The boundary conditions to be imposed on the solution to Eqs. (1) and (2) are as follows. At the body surface $y = 0$, where $u = v = 0$ and $T = T_w$, and either $\alpha(x, 0) = 0$ for a catalytic wall (complete atom recombination) or $\partial\alpha/\partial y = 0$ for a noncatalytic wall (no atom recombination and diffusion at the surface). At the outer edge of the boundary layer $y \to \infty$, neglecting vorticity interaction effects, $u = u_e(x)$, $T = T_e(x)$, and $\alpha = \alpha_e(x)$, where

$$\frac{d\alpha_e}{dx} = -\frac{4k_R{}'T_e{}^{\omega-2}}{u_e}\cdot g_e\left(\frac{p_e}{\mathcal{R}}\right)^2 \tag{3}$$

Equation (3) expresses the balance between convective and reactive composition change along the streamlines at the edge of the boundary layer in the assumed absence of vorticity interaction effects. This equation accounts for the explicit effect of reaction in the inviscid flow and must be included in the description of the nonequilibrium boundary layer to insure that the outer boundary condition is compatible with the inviscid flow composition gradient. For an equilibrium-dissociated inviscid flow $[g_e(x) = 0]$, the coefficient of g_e is regarded as infinite,† whereas in the opposite extreme of a chemically frozen flow ($d\alpha_e/dx = 0$), this coefficient is taken to be negligible, and g_e is generally not zero.

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† At the stagnation point of a symmetrical blunt body, g_e is always zero (and α_e is always constant) provided that low Reynolds number and viscous shock layer phenomena are neglected.

It is convenient to express Eq. (1) in terms of the familiar similarity coordinates ξ,η of boundary-layer theory, since the various boundary-layer profiles involved are fairly well known or can easily be estimated from solutions available in the literature. Thus, defining a stream function $f(\xi,\eta)$ such that $\partial f/\partial\eta \equiv f' = u/u_e$, applying the transformation

$$\xi \equiv \int_0^x \rho_e\mu_e u_e r_B{}^{2\epsilon}\,dx \qquad \eta \equiv \frac{r_B{}^{\epsilon}u_e}{(2\xi)^{1/2}}\int_0^y \rho dy \tag{4}$$

$\epsilon = 0$, two-dimensional
$\epsilon = 1$, axisymmetric

and introducing the new dependent variables $z = \alpha/\alpha_e$, $\theta = T/T_e$, and $\kappa = x(d\ln\xi/dx)$, one obtains the following generalized species conservation equation for a nonequilibrium-dissociated boundary layer with a simultaneously reacting nonequilibrium inviscid flow:

$$\alpha_e\left[f\frac{\partial z}{\partial\eta} + \frac{\partial^2 z}{\partial\eta^2} - 2\xi\left(f'\frac{\partial z}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial z}{\partial\eta}\right)\right] = \Gamma\left[\frac{\alpha_e{}^2}{1+\alpha_e}G(z,\theta) - g_eH(z,\theta)\right] \tag{5}$$

where

$$G(z,\theta) \equiv \theta^{\omega-2}\left\{\left(\frac{1+\alpha_e}{1+\alpha_e z}\right)z^2 - \left(\frac{1-\alpha_e z}{1-\alpha_e}\right)\exp\left[-\frac{\theta_D}{\theta}(1-\theta)\right]\right\} \tag{6}$$

$$H(z,\theta) \equiv f'z - \left(\frac{1-\alpha_e z}{1-\alpha_e}\right)\theta^{\omega-2}\exp\left[-\frac{\theta_D}{\theta}(1-\theta)\right] \tag{7}$$

and

$$\Gamma \equiv \frac{8k_R{}'T_e{}^{\omega-2}x}{\kappa u_e}\left(\frac{p_e}{\mathcal{R}}\right)^2 \tag{8}$$

is a characteristic (flow time/reaction time) ratio originally defined by Fay and Riddell.[1] It should be noted that in deriving Eq. (5), the inviscid flow relation $\Gamma g_e = -d\alpha_e/d\ln\xi$ from (3) was employed, and the factor $\exp A/4p_e$ in the dissociation term of Eq. (2) eliminated in favor of the variables α_e, T_e, and g_e. In addition, one has the outer boundary conditions $f'(\xi,\infty) = \theta(\xi,\infty) = z(\xi,\infty) = 1$, whereas at the surface one has $f(\xi,0) = f'(\xi,0) = 0$ and $\theta(\xi,0) = \theta_w(\xi)$, and either $z(\xi,0) = 0$ for a catalytic wall or $(\partial z/\partial\eta)(\xi,0) = 0$ for a perfectly noncatalytic wall.

Now, the two reaction terms on the right-hand side of Eq. (5) may be identified as follows. The first reaction term, $[\alpha_e{}^2/(1+\alpha_e)]\Gamma G$, which involves contributions from the recombination ($\backsim z^2$) and dissociation rates, represents the net reaction rate distribution across the boundary layer *if there were an equilibrium-dissociated inviscid flow.* This term is precisely that involved in the analysis of Fay and Riddell and is also equivalent to the reaction rate term used by other investigators up to the present time. The function G vanishes identically for an equilibrium boundary layer; it is not zero, however, in the opposite extreme of frozen flow. The second reaction term, $g_e\Gamma H$, represents the additional effect of nonequilibrium chemical reaction (streamwise atom concentration gradient) in the inviscid flow on the boundary layer. The function $f'z$ in this term constitutes the convective change of the inviscid flow composition, continued into the boundary layer and modified by the decreasing streamwise velocity therein. Similarly, the exponential factor is the inviscid flow dissociation rate as modified by the boundary-layer temperature field. It will be the purpose of this paper to evaluate the relative significance of these two reaction rate terms for various nonequilibrium flow situations encountered in practice.

Some typical profiles of G and H across a dissociated boundary layer for the extreme conditions of either frozen or equilibrium boundary-layer flow (which give the maximum and minimum values, respectively, of these two functions) are presented in Figs. 1 and 2. These profiles were computed using the Blasius function for $f'(\eta)$, the generalized Crocco energy equation integral

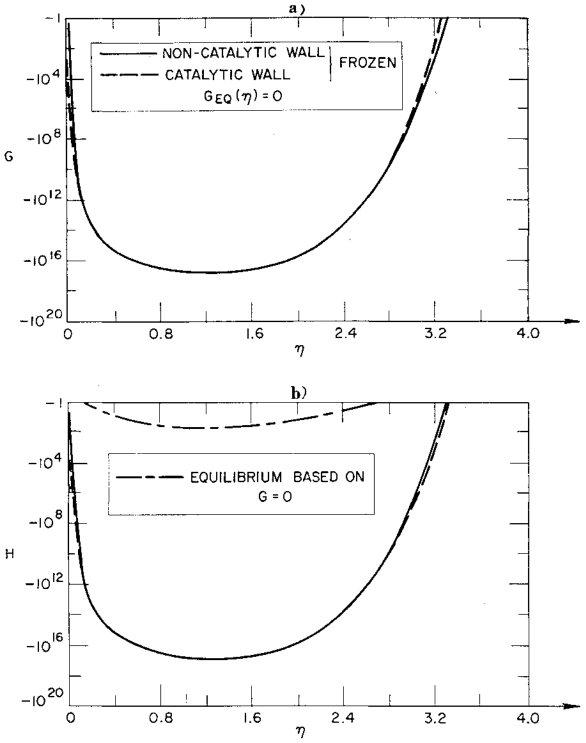

Fig. 2 Reaction rate distribution functions G and H for highly dissipative flow; $\theta_w = 1$, $U_e{}^2/2\bar{c}_pT_e = 50$, $\theta_D = 50$, $\alpha_e = 0.01$, $P_R = L_e = 1$.

$$\theta = \theta_w + (1-\theta_w)f' + f'(1-f')\left(\frac{u_e{}^2}{2\bar{c}_pT_e}\right) - \frac{\alpha_e h_D}{\bar{c}_pT_e}\{z - z(0) - f'[1 - z(0)]\} \tag{9}$$

and a self-similar atom concentration profile given either by the frozen flow solution to Eq. (5)[8] or by a simultaneous solution of the equilibrium relation $g(\alpha, p_e, T) = 0$ and Eq. (9).‡ In the case of a highly cooled flow with negligible viscous dissipation shown in Fig. 1, where gas phase recombination near the body surface is the controlling reaction rate,[4, 5, 9] it is seen that $G \gg H$ throughout the inner half of the boundary layer, except when the boundary layer approaches equilibrium. The ratio $G_{\max}/H_{\max}$, however, depends strongly on the surface catalycity, decreasing by four orders of magnitude from a noncatalytic to a completely catalytic surface condition, although still remaining quite small in the latter case.§ On the other hand, in the case of flows in which viscous heating controls the temperature profile in the boundary layer (Fig. 2), G and H are both dominated by the large exponential dissociation rate term so that [as can be seen from Eq. (7)] $H \approx G \ll -1$ throughout the boundary layer. In contrast to the highly cooled, nondissipative case, both G and H are relatively insensitive to the heterogeneous

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‡ Strictly speaking, the calculation of H using boundary-layer solutions in which the effect of inviscid flow reaction has been neglected is not rigorously consistent; however, for the present purpose of estimating the magnitude of H relative to G, this procedure is acceptable.

§ The values of $G_{\max}$ shown in Fig. 1 were obtained by neglecting the exponential dissociation term in Eq. (6), using Eq. (9), and employing the frozen flow solutions $z = 1$ and $z = f'$ for a noncatalytic or catalytic wall, respectively.

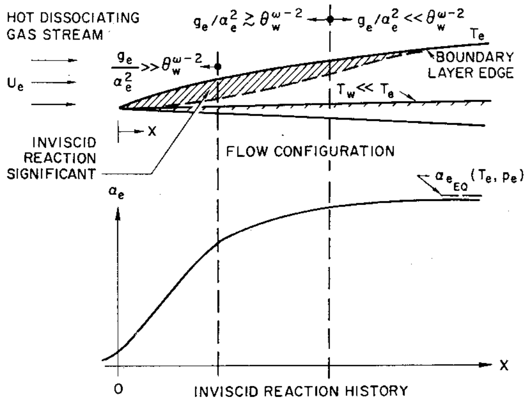

Fig. 3 Freestream dissociation effect on a flat plate boundary layer.

reaction rate on the surface. It can be generally noted from these examples that $H(\eta)$ does not vanish under the usually assumed conditions for an equilibrium boundary layer [$\Gamma \to \infty, G(\eta) \to 0$]. Thus, when $g_e \neq 0$, Eq. (5) yields

$$G(\eta)_{\Gamma\to\infty} \simeq (g_e/\alpha_e^2)(1+\alpha_e)H_{\mathrm{eq}}(\eta) \qquad (10)$$

which implies, as expected, that the boundary layer cannot actually attain equilibrium throughout unless the inviscid flow has also reached equilibrium. However, since $H_{\mathrm{eq}}(\eta)$ remains very small in comparison to H_F (especially for highly dissipative flows) and $G_F \gtrsim H_F$ (Fig. 2), $G_{\Gamma\to\infty}$ is negligible in comparison to G_F unless $g_e/\alpha_e^2 \gg 1$ for a highly cooled flow. As will be shown below, the conditions under which the latter situation occurs in practice favor the existence of a nearly frozen boundary layer.

Appraisal of Inviscid Flow Reaction Effect

The typical distributions of G and H having been established, the effect of the latter function on a nonequilibrium boundary layer can now be assessed by examining the coefficients of these functions in Eq. (5).

Highly-Cooled Flows

Since $G \gg H$ throughout the inner half of the nonequilibrium boundary layer in such flows, it is clear from Eq. (5) and Fig. 1 that the effect of inviscid flow reaction on the boundary layer for a given value of Γ may be neglected except near the outer edge ($\eta \gg 1$), provided that

$$\left(\frac{1+\alpha_e}{\alpha_e^2}\right) g_e \ll \theta_w^{\omega-2} \qquad \text{noncatalytic wall} \quad (11a)$$

$$\left(\frac{1+\alpha_e}{\alpha_e^2}\right) g_e \ll 10^{-2}(0.91\theta_w + 0.09)^{\omega-2} \qquad \text{catalytic wall} \quad (11b)$$

These conditions imply that gas phase recombination near the wall completely controls the chemical state throughout much of the boundary layer and the inviscid flow reaction is so slow by comparison that it may be considered chemically frozen. This situation applies, for example, to the nose region of highly cooled hypersonic blunt bodies at high altitudes, where a substantial departure from equilibrium can occur in both the inviscid and viscous portions of the flow downstream of the sonic line.[2, 10] Here, g_e/α_e^2 is usually on the order of unity or less, so that the effects of simultaneous reaction in the inviscid flow *as manifested in the function H* may be neglected in computing the overall nonequilibrium boundary-layer properties (such as heat transfer) around the nose when the body is highly cooled. This assumption of a frozen inviscid flow was taken a priori in Ref. 2.

Another situation of interest arises in connection with the boundary layer in the nozzle of an advanced hypersonic testing facility, where a significant departure from equilibrium in the core of dissociated inviscid flow can occur downstream of the throat. Here, nonequilibrium reaction will take place in both the inviscid and viscous portions of the flow in the region of moderately supersonic flow and wall cooling, where the inviscid flow is changing from equilibrium to frozen. This problem has recently been studied numerically by Brainerd and Levinsky[11] for the case of air with coupled, finite rate reactions. Using local similarity, these authors match the boundary-layer solution to the reacting inviscid flow solution taking into account the streamwise concentration gradient terms. Unfortunately, little can be observed directly from the numerical results in Ref. 11 concerning the effect of the inviscid reaction rate alone on the boundary layer, since only the *total* effects of reaction in both the inviscid flow and the boundary layer are presented. However, these results do indicate that the effect of gas phase reaction on the nonequilibrium boundary layer downstream of the throat is very sensitive to the surface catalycity of the nozzle wall. Since the effect of the inviscid flow concentration gradients on the nonequilibrium boundary layer is comparatively insensitive to the surface catalycity for a highly cooled wall (Fig. 1), the pronounced effect of surface catalycity found by Brainerd and Levinsky clearly implies that gas phase recombination within the boundary layer near the wall is by far the most predominant reaction rate. Furthermore, this agreement with the present analysis has been more directly confirmed by additional, unpublished calculations for the nozzle problem,[13] which show that the inviscid composition gradient term has a noticeable effect on the thermodynamic-state profiles only near the outer edge of the boundary layer.

An exceptional situation can arise in the special case of a cooled body such as a plate or wedge immersed in a slow, hot, dissociatively relaxing gas stream (Fig. 3). Here, the inviscid reaction effect depends strongly on the state of relaxation in the incoming stream as well as the surface temperature and catalycity. Generally speaking, the inviscid flow must be close to equilibrium before Eqs. (11) can be satisfied in this case; otherwise, the exponential dissociation term in g_e can be much larger than $\theta_w^{\omega-2}$ even for a noncatalytic wall with $\theta_w \leq 0.10$ and $\omega = -1.5$. Therefore, when the incoming gas is in a highly nonequilibrium state, both reaction terms in Eq. (5) can be important over a significant extent of the body as illustrated in Fig. 3. In fact, in the leading edge region, the inviscid dissociation rate can conceivably be much larger than the boundary-layer recombination rate near the surface ($g_e/\alpha_e^2 \gg \theta_w^{\omega-2}$). Except near the outer edge, the boundary layer in this region may therefore be considered frozen with respect to the inviscid flow and the reaction term involving G neglected altogether. This situation has been treated recently by Chung[12] for the analogous unsteady problem of the wall boundary-layer development in the dissociating gas flow behind a strong shock in a shock tube. He found that the function G can be neglected in comparison to H until the relaxing inviscid flow is almost completely equilibrated. It may be noted that Chung further simplified the analysis by neglecting the exponential part of the function H, taking $H \simeq f'z$. As may be seen from Fig. 1, such an approximation is quite accurate in the inner one-half of a highly cooled boundary layer. Moreover, since it is the values of H for $\eta \lesssim 1$ that primarily determine the surface properties, this approximation appears to be a satisfactory one for the calculation of heat transfer, provided viscous dissipation in the boundary layer is negligible.

Flows with Large Viscous Heating

Significant heating within the boundary layer due to viscous dissipation is encountered, for example, in hypersonic flows over slender aerodynamic bodies such as cones. Here, inviscid flow reaction may arise because of the presence of a highly dissociated, nonequilibrium freestream such as created in an advanced hypersonic wind tunnel nozzle. Alternatively, one may envisage the case of hypersonic flow over a moderately slender cone where the temperature rise across the shock wave is large enough to excite a significant dissociation rate in the local inviscid flow. (This may also occur due to boundary-layer-induced pressure and temperature rise in the strong interaction near the leading edge of a plate or very slender cone in hypersonic flow.) Since $H \simeq G$ in such cases (Fig. 2), except when the boundary layer is near equilibrium, the reaction terms on the right-hand side of Eq. (5) may be combined into the approximation

$$\Gamma\left(\frac{\alpha_e^2}{1+\alpha_e} G - g_e H\right) \simeq \Gamma G \frac{(1-\alpha_e)\exp(A-\theta_D)}{4p_e} \qquad g_e \text{ arbitrary} \qquad (12)$$

using the definition of g_e. However, it may be readily seen that the same result is obtained if the local inviscid flow is assumed to be in equilibrium a priori ($g_e = 0$). Therefore, as far as the behavior of the nonequilibrium boundary layer in the similarity variables $z(\xi,\eta)$ and $\theta(\xi,\eta)$ is concerned, the contribution of the function H due to nonequilibrium inviscid flow reaction can be neglected. That is, the error involved in the reaction rate approximation equation (12) for $g_e \neq 0$, which Eq. (5) shows to be

$$\Gamma\left(\frac{\alpha_e^2}{1+\alpha_e z} \theta^{\omega-2} z^2 - g_e f' z\right) \qquad (13)$$

is always very small in comparison to Eq. (12), regardless of Γ, for flows involving large viscous dissipation within the boundary layer.

Concluding Remarks

It is concluded from the present study that the effect of inviscid flow nonequilibrium reaction, introduced by the streamwise composition gradient, can usually be neglected regardless of the surface catalycity, unless the finer details of the thermodynamic-state profiles near the outer edge of the boundary layer are of interest. An exception was observed in the special case of a highly cooled body immersed in a slow, hot, dissociatively relaxing gas stream, where the inviscid reaction rate can be comparable to or even much greater than the viscous flow reaction rate near the leading edge of the body.

It should be noted that these conclusions by no means imply that the chemical state of the inviscid flow has a negligible effect on the boundary layer; indeed, this state can have an important effect on the values of α_e, T_e, and Γ appearing in the boundary-layer equations. Rather, the present analysis shows that the failure to properly match the viscous flow reaction rate at the outer edge of the boundary layer to the inviscid flow rate will usually cause but a small error in solving the boundary-layer equations.

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